Trigonometry Workbook Solutions

Angles and their measure

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MATH

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

■ 1. Find the supplement θ of $7\pi/8$ radians.

Solution:

The angle θ and the angle $7\pi/8$ radians are supplementary, which means their sum is π radians.

$$\theta + \frac{7\pi}{8} = \pi$$

$$\theta = \pi - \frac{7\pi}{8}$$

$$\theta = \frac{8\pi}{8} - \frac{7\pi}{8}$$

$$\theta = \frac{\pi}{8} \text{ radians}$$

■ 2. Angles $\angle 1$ and $\angle 2$ are complementary. Find the supplement of $\angle 2$.

$$m\angle 1 = (2x + 5)^\circ$$

$$m\angle 2 = (x + 4)^\circ$$

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Solution:

Because $\angle 1$ and $\angle 2$ are complementary, that means they sum to $90°$.

$$m\angle 1 + m\angle 2 = 90°$$

$$2x + 5 + x + 4 = 90°$$

$$3x + 9 = 90°$$

$$3x = 81$$

$$x = 27$$

Plugging this back into $\angle 2$ gives

$$m\angle 2 = (27 + 4)° = 31°$$

The supplement of $\angle 2$ is therefore

$$180° - 31°$$

$$149°$$

■ 3. The complement of θ is $\pi/6$ radians. Find the supplement of θ.

Solution:

Supplementary angles sum to $\pi/2$ radians. So

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$$\theta + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\theta = \frac{3\pi}{6} - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}$$

Sine supplementary angles sum to π radians, the supplement of θ is

$$\pi - \frac{\pi}{3}$$

$$\frac{3\pi}{3} - \frac{\pi}{3}$$

$\frac{2\pi}{3}$ radians

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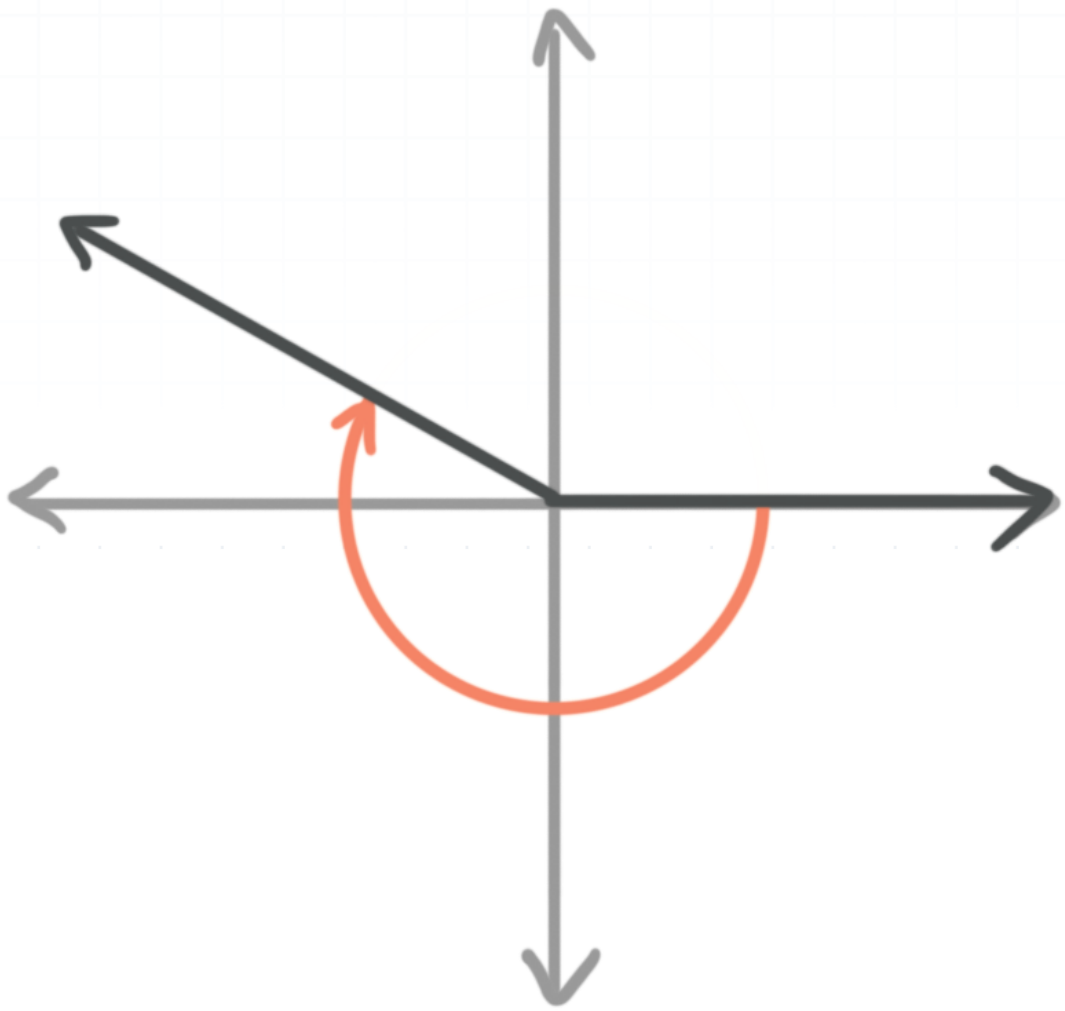
SKETCHING ANGLES IN STANDARD POSITION

■ 1. Sketch -210° in standard position.

Solution:

The terminal side of the angle is found by rotating 210° in a negative (clockwise) direction, so the terminal side lies in quadrant II.



■ 2. Find the measure of the angle (in radians).

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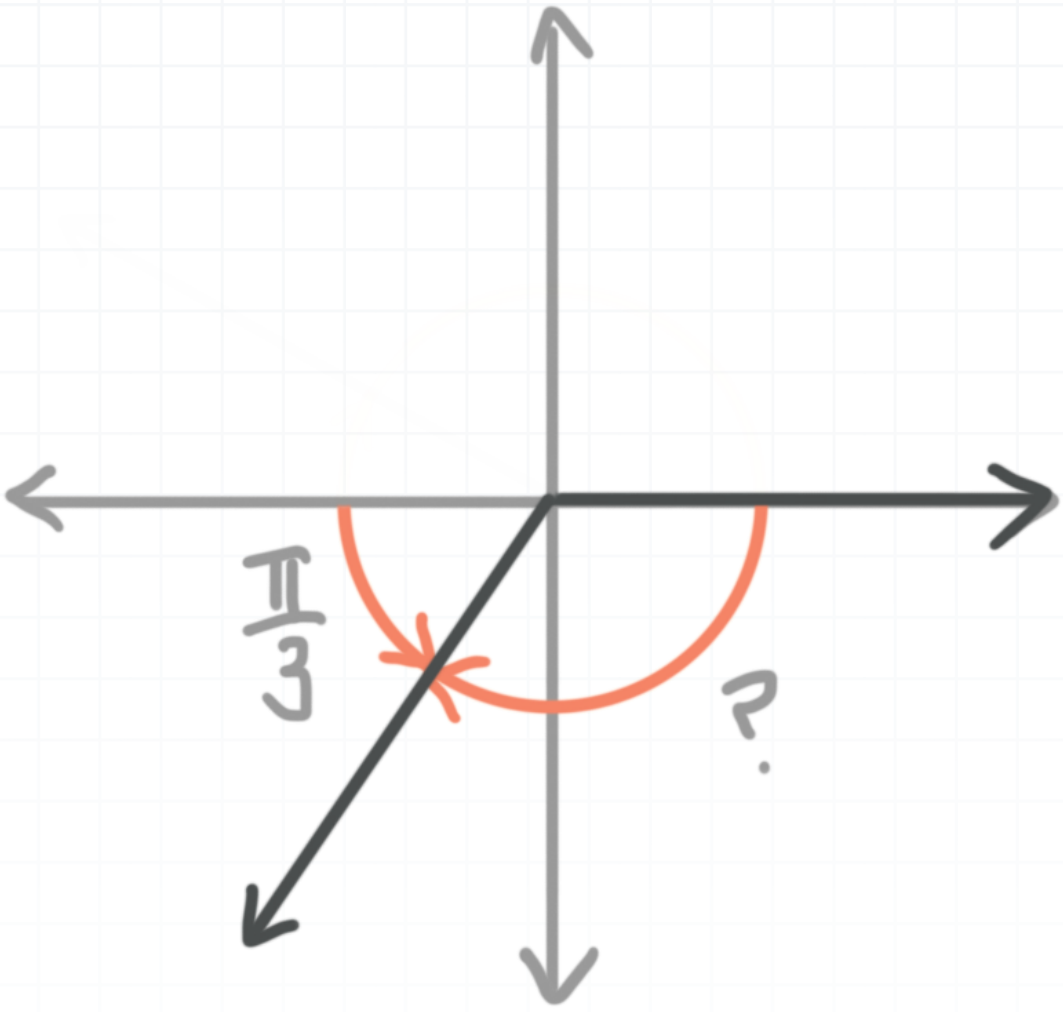

Solution:

The two angles shown are supplementary, which means they sum to π radians. Which means the measure of the unknown angle is

$$\pi - \frac{\pi}{3}$$

$$\frac{3\pi}{3} - \frac{\pi}{3}$$

$$\frac{2\pi}{3}$$

Because the angle shows clockwise rotation, that means it's the negative angle

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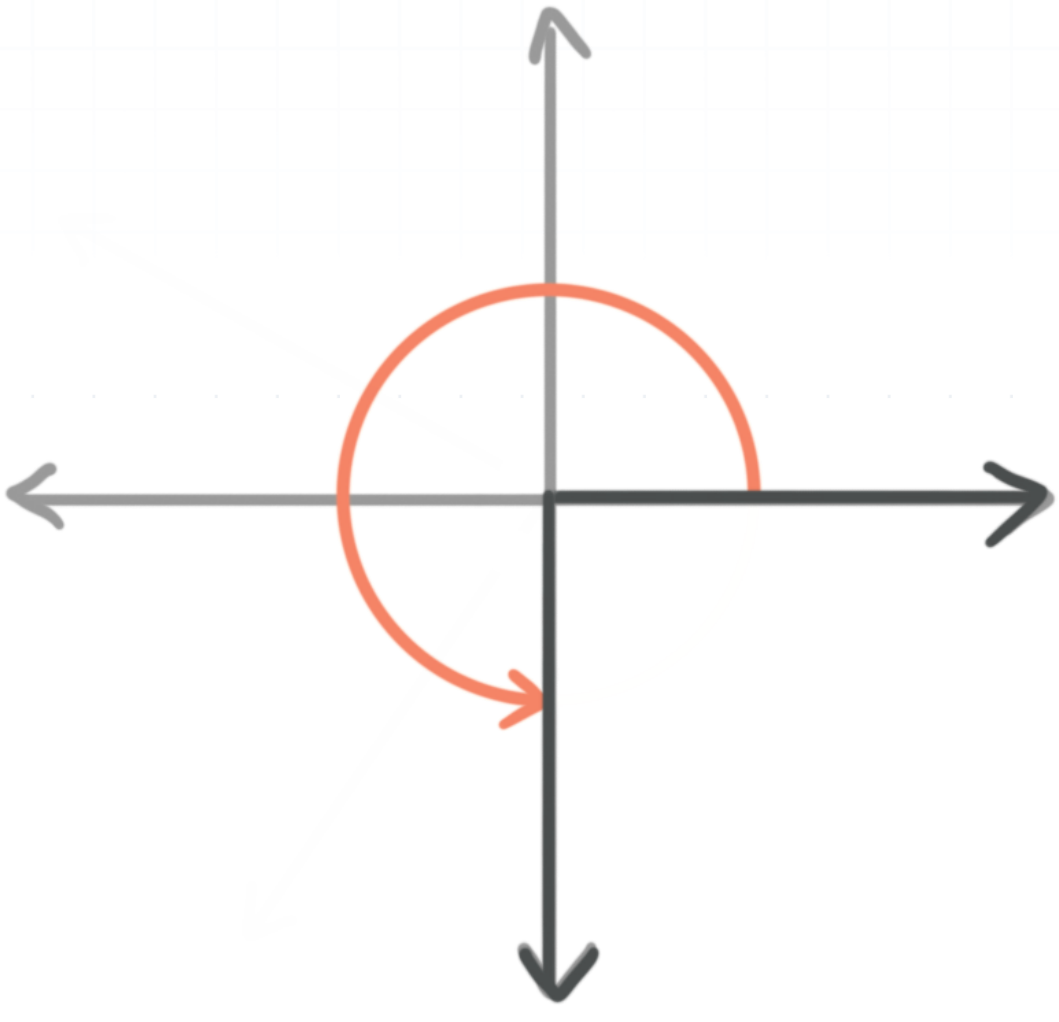
$$-\frac{2\pi}{3}$$

■ 3. Sketch the angle in standard position.

$\frac{3\pi}{2}$ radians

Solution:

A sketch of the angle is



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QUADRANT OF THE ANGLE

■ 1. In which quadrant is the angle located?

$\frac{\pi}{5}$ radians

Solution:

The quadrants are divided by intervals of $\pi/2$, so let's convert the angle into one with a denominator of 2.

$$\pi \cdot \frac{2}{5}$$

$$5 \cdot \frac{2}{5}$$

$$0.4 \cdot \frac{\pi}{2}$$

Since the first quadrant is bounded by $0 \cdot \pi/2$ and $1 \cdot \pi/2$, and because $0 < 0.4 < 1$, we know $\pi/5$ has to lie in the first quadrant.

■ 2. In which quadrant is the angle located?

-820°

Solution:

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If we add multiples of $360°$ to $-820°$, we'll eventually get back to an angle that we recognize on the interval $[0,2\pi)$.

$$-820° + 360° = -460°$$

$$-460° + 360° = -100°$$

$$-100° + 360° = 260°$$

Since the third quadrant is bounded by $180°$ and $270°$, and $180° < 260° < 270°$, we know $-820°$ has to lie in the third quadrant.

■ 3. In which quadrant is the angle located?

$$-\frac{13\pi}{4} \text{ radians}$$

Solution:

If we add multiples of 2π to $-13\pi/4$, we'll eventually get back to an angle that we recognize on the interval $[0,2\pi)$.

$$-\frac{13\pi}{4} + 2\pi = -\frac{5\pi}{4}$$

$$-\frac{5\pi}{4} + 2\pi = \frac{3\pi}{4}$$

Since the second quadrant is bounded by $2\pi/4$ and $4\pi/4$, and because

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$$\frac{2\pi}{4} < \frac{3\pi}{4} < \frac{4\pi}{4}$$

we know $3\pi/4$ has to lie in the second quadrant.

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FINDING COTERMINAL ANGLES

■ 1. If you start with $-270°$ and rotate it three times clockwise around the origin, find the measure of the new angle.

Solution:

If we call the new angle α, then the measure of the new angle will be

$$\alpha = -270° + (-3)(360°)$$

$$\alpha = -270° - 1{,}080°$$

$$\alpha = -1{,}350°$$

■ 2. If you start with $5\pi/6$ radians and rotate it two times counterclockwise around the origin, find the measure of the new angle.

Solution:

If we call the new angle α, then the measure of the new angle will be

$$\alpha = \frac{5\pi}{6} + 2(2\pi)$$

$$\alpha = \frac{5\pi}{6} + 4\pi$$

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$$\alpha = \frac{5\pi}{6} + \frac{24\pi}{6}$$

$$\alpha = \frac{29\pi}{6} \text{ radians}$$

■ 3. Find the negative and positive coterminal angles that are one rotation around the origin from $\theta = 200°$.

Solution:

To find angles that are coterminal with $\theta = 200°$, we need to add and subtract $360°$. So the coterminal angles that are one full rotation from $\theta = 200°$ are

$$200° + (-1)(360°) = -160°$$

$$200 + (1)(360°) = 560°$$

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CONVERTING BETWEEN DEGREES AND DMS

■ 1. If the measure of an angle is given in DMS, find its measure in decimal degrees.

$$65°13'12''$$

Solution:

The degrees part of $65°13'12''$ is $65°$, so (in decimal degrees) the integer part of its measure is $65°$. The minute part is

$$13' = 13'(1)$$

$$13' = 13'\left(\frac{1°}{60'}\right)$$

$$13' = \frac{13°}{60}$$

The second part is

$$12'' = 12''(1)$$

$$12'' = 12''\left(\frac{1°}{60'}\right)\left(\frac{1'}{60''}\right)$$

$$12'' = \frac{12}{(60)(60)} = \frac{12}{3600} = \frac{1}{300}°$$

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So

$$13'12'' = 13' + 12'' = \frac{13}{60}^{\circ} + \frac{1}{300}^{\circ} = 0.22^{\circ}$$

When we include the integer part of the angle, we get

$$65^{\circ}13''' = 65^{\circ} + 0.22^{\circ} = 65.22^{\circ}$$

■ 2. Convert $-172^{\circ}4'36''$ to decimal degrees.

Solution:

Break up each part of the DMS angle, and multiply by conversion factors.

$$-172^{\circ} + 4' + 36''$$

$$-172^{\circ} + 4'\left(\frac{1^{\circ}}{60'}\right) + 36''\left(\frac{1^{\circ}}{60'}\right)\left(\frac{1'}{60''}\right)$$

$$-172^{\circ} + \frac{4}{60}^{\circ} + \frac{36}{3{,}600}^{\circ}$$

$$-172^{\circ} + \frac{276}{3{,}600}^{\circ}$$

$$-172.0767^{\circ}$$

■ 3. Find the sum in DMS.

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$$20.25^\circ + 20^\circ 2' 5''$$

Solution:

Convert 20.25° to DMS.

$$20.25^\circ$$

$$20^\circ + 0.25^\circ$$

$$20^\circ + \frac{25}{100}^\circ \left(\frac{60'}{1^\circ}\right)$$

$$20^\circ + 15'$$

$$20^\circ 15'$$

Then find the sum.

$$20.25^\circ + 20^\circ 2' 5''$$

$$20^\circ 15' + 20^\circ 2' 5''$$

$$20^\circ + 20^\circ + 15' + 2' + 5''$$

$$40^\circ + 17' + 5''$$

$$40^\circ 17' 5''$$

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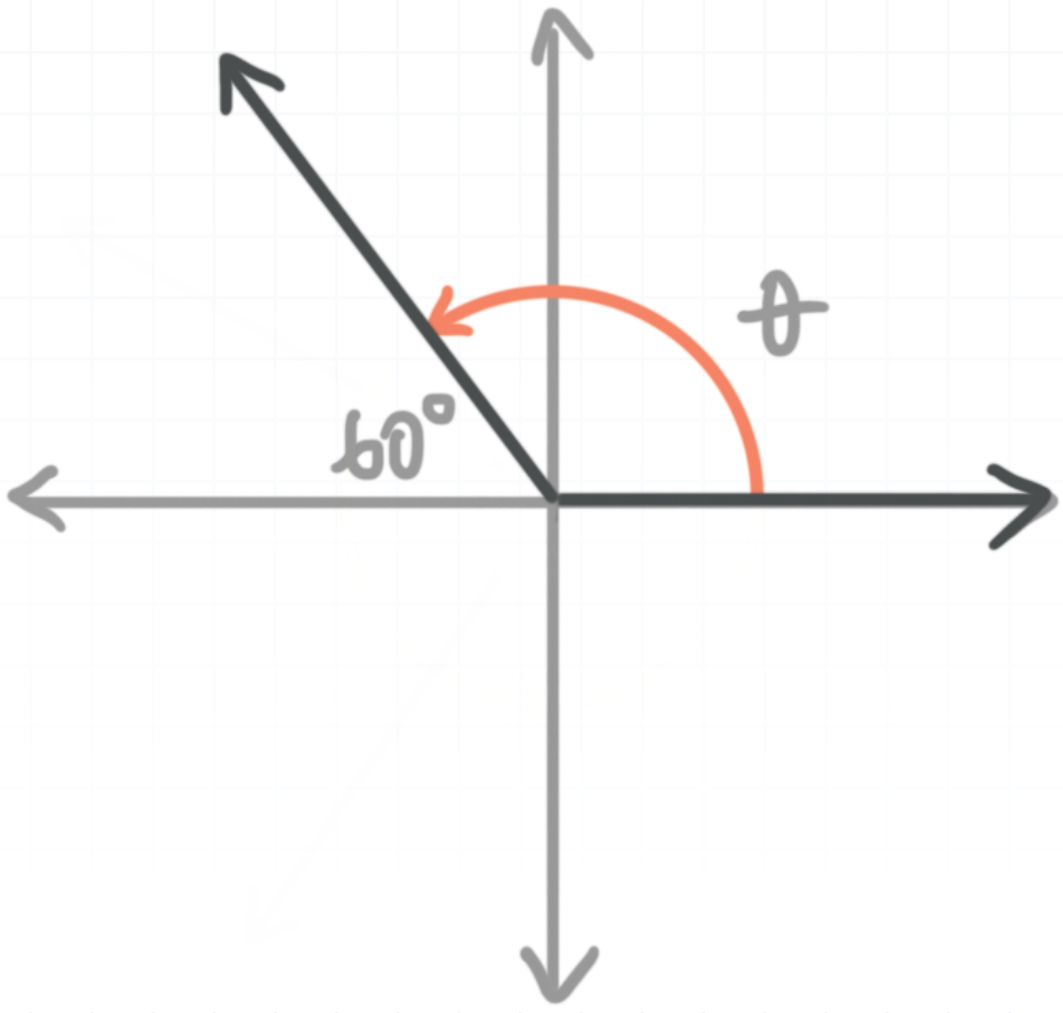
CONVERTING BETWEEN DEGREES AND RADIANS

1. What is the measure (in radians) of the angle θ ?



Solution:

The angle has a measure of

$$180^\circ - 60^\circ$$

$$120^\circ$$

Convert this to radians.

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$$120^\circ\left(\frac{\pi}{180^\circ}\right)$$

$\frac{2\pi}{3}$ radians

■ 2. Convert the angle measure to degrees.

$-\frac{9\pi}{2}$ radians

Solution:

Convert the angle measure to degrees.

$$-\frac{9\pi}{2}\left(\frac{180^\circ}{\pi}\right)$$

-810°

■ 3. What is the best approximation (in degrees) of the angle?

2.36 radians

Solution:

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$$2.36 \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}} \right)$$

$$2.36 \left(\frac{180^\circ}{\pi} \right)$$

$$2.36 \left(\frac{180^\circ}{\pi} \right)$$

$$135.2^\circ$$

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FUNCTIONS OF NEGATIVE ANGLES

1. Find the angle that's coterminal with $-748°$ and lies in the interval $[-90°,90°]$.

Solution:

We need the angle $-748°$ plus some multiple of $360°$ to lie inside $[-90°,90°]$, so we can set up the inequality

$$-90° \leq \theta + 360°n \leq 90°$$

where n is an integer.

$$-90° \leq -748° + 360°n \leq 90°$$

$$-90° + 748° \leq 360°n \leq 90° + 748°$$

$$658 \leq 360°n \leq 838°$$

$$1.83 \leq n \leq 2.33$$

Since n has to be an integer, $n = 2$, and the coterminal angle we're looking for is

$$\theta = -748 + 2(360°)$$

$$\theta = -28°$$

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■ 2. Find the angle that's coterminal with $-21\pi/4$ radians and lies in the interval $[-4\pi, -2\pi]$.

Solution:

We need the angle $-21\pi/4$ plus some multiple of 2π to lie inside $[-4\pi, -2\pi]$, so we can set up the inequality

$$-4\pi \leq -\frac{21\pi}{4} + n2\pi \leq -2\pi$$

where n is an integer.

$$-4\pi + \frac{21\pi}{4} \leq n2\pi \leq -2\pi + \frac{21\pi}{4}$$

$$-\frac{16\pi}{4} + \frac{21\pi}{4} \leq n2\pi \leq -\frac{8\pi}{4} + \frac{21\pi}{4}$$

$$\frac{5\pi}{4} \leq n2\pi \leq \frac{13\pi}{4}$$

$$\frac{5\pi}{8\pi} \leq n \leq \frac{13\pi}{8\pi}$$

$$\frac{5}{8} \leq n \leq \frac{13}{8}$$

$$0.625 \leq n \leq 1.625$$

Since n has to be an integer, $n = 1$, and the coterminal angle we're looking for is

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$$\theta = -\frac{21\pi}{4} + (1)(2\pi)$$

$$\theta = -\frac{21\pi}{4} + \frac{8\pi}{4}$$

$$\theta = -\frac{13\pi}{4}$$

■ 3. θ has a measure of $42°$. If the angle is rotated 6 full rotations in the negative direction, find its measure after the rotations.

Solution:

The new angle measure will be given by

$$42° + (-6)(360°)$$

$$42° - 2{,}160°$$

$$-2{,}118°$$

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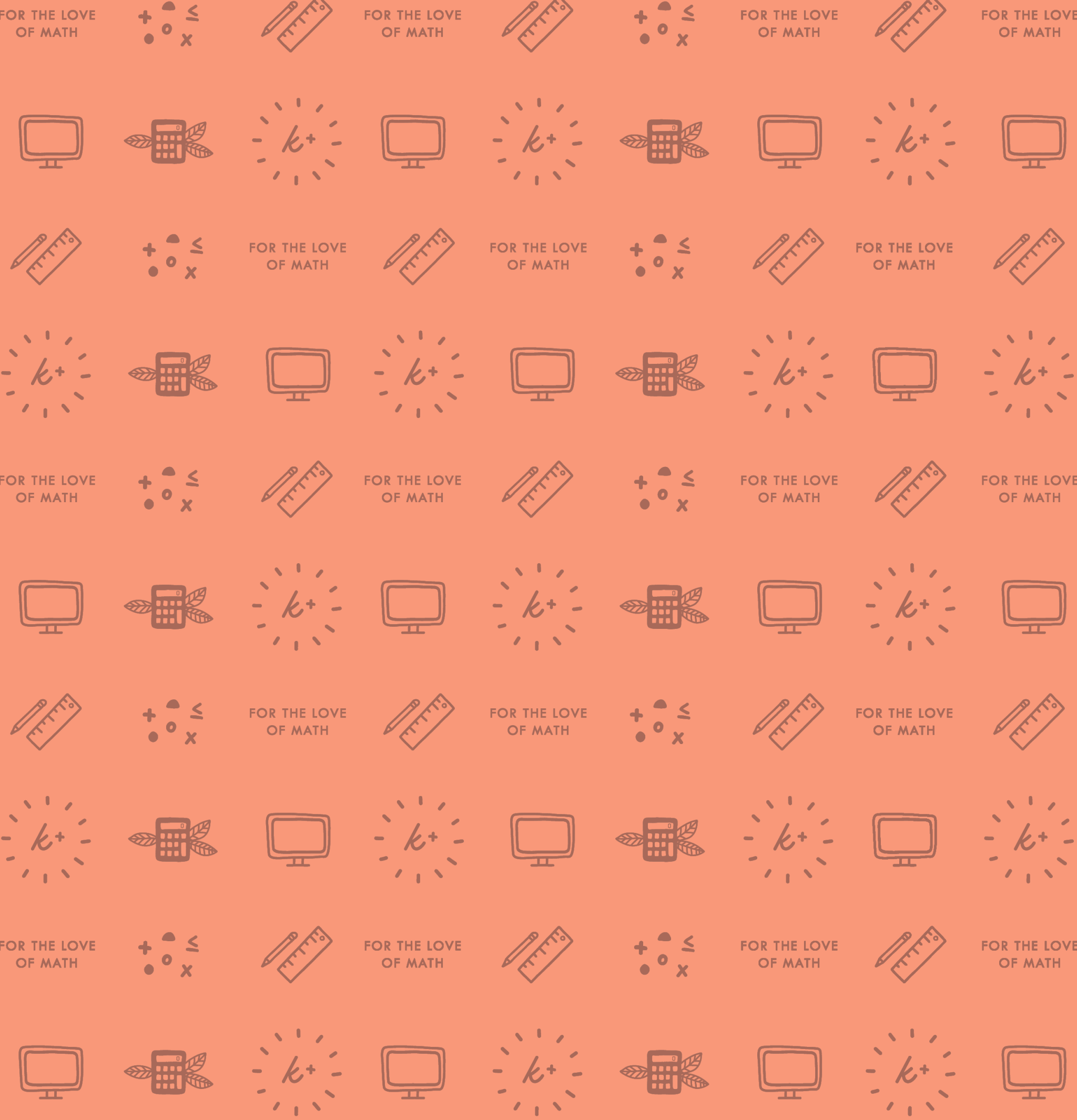
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